UNCLASSIFIED

THE ANTENNA LABORATORY

RESEARCH ACTIVITIES in ---

Automatic Controls	Antennas	Echo Area Studies
Microwave Circuits	Astronautics	E M Field Theory
Terrain Investigations	Radomes	Systems Analysis
Wave Propagation		Submillimeter Applications

AFCRL 384(I)

Some Principles of Low-Noise Antenna Design
(Final Engineering Report, Vol. I)
1 December 1959 through 30 November 1960

by

Ross Caldecott

Contract AF 19(604) -6134

1041-1 1 December 1960

Department of ELECTRICAL ENGINEERING

THE OHIO STATE UNIVERSITY
RESEARCH FOUNDATION
Columbus , Ohio

Best
Available
Copy

NOTICES

Requests for additional copies by Agencies of the Department of Defense, their contractors, and other Government agencies should be directed to the:

ARMED SERVICES TECHNICAL INFORMATION AGENCY
ARLINGTON HALL STATION
ARLINGTON 12, VIRGINIA

Department of Defense contractors must be established for ASTIA services or have their "need-to-know" certified by the cognizant military agency of their project or contract.

All other persons and organizations should apply to the:

U.S. DEPARTMENT OF COMMERCE
OFFICE OF TECHNICAL SERVICES
WASHINGTON 25, D.C.

REPORT

by

THE OHIO STATE UNIVERSITY RESEARCH FOUNDATION

COLUMBUS 12, OHIO

Cooperator	Electronics Research Directorate Air Force Cambridge Research Laboratories Office of Aerospace Research Unites States Air Force Bedford, Massachusetts
Contract	AF 19(604)-6134
Investigation of	Investigation of Low-Noise Antenna Systems
Subject of Report	Some Principles of Low-Noise Antenna Design (Final Engineering Report, Vol. I) 1 December 1959 through 30 November 1960
Submitted by	Ross Caldecott Antenna Laboratory Department of Electrical Engineering
Date	1 December 1960

ABSTRACT

The principal sources of antenna noise are discussed together with their effects on antenna performance and methods of reducing these effects. The concept of antenna gain-temperature ratio is introduced and a mathematical method of handling it as a single design parameter evolved. Some limitations of very large continuous apertures are considered and a multielement system proposed as a means of increasing the useful size of large antennas. Some experiments to determine the feasability of such a system and the optimum size of its elements are described.

LIST OF RELATED PUBLICATIONS AND PAPERS

1. Hame, T.G., Eberle, J.W., Caldecott, R., and Taylor, R.C., "High Gain and Low Noise Antenna Research", Tenth Annual Symposium on The USAF Antenna Research and Development Program. University of Illinois, October 1960.

2. Caldecott, R. and Peake, W.H., "Some Aspects of Low Noise Antenna Design," Symposium on the Application of Low Noise Receivers to Radar. M.I.T., Lincoln Lab., October 1960.

3. Caldecott, R. and Peake, W.H., "Designing Low Noise Antennas" Electronics, 34 No. 3, 1961.

4. Caldecott, R. and Eberle, J.W., and Hame, T.G., "Optimizing the Performance of Very High Gain Low Noise Antennas". To be presented at the 1961 IRE International Convention in New York.

GLOSSARY OF SYMBOLS USED

A	=	Attenuation constant .
a	=	Semi-beam width of feed radiation pattern .
B	=	Bandwidth (cps) .
b	=	Half-angle subtended by parabaloid at the focus .
c	=	Power coupling factor.
D	=	Directivity of antenna relative to arbitrary level .
D'	=	Value of D for uniform illumination .
D_o	=	Directivity of antenna relative to uniformly illuminatad case .
E	=	Efficienty of antenna.
e	=	Exponential operator .
F	=	Feed field stregth in direction θ .
G	=	Antenna gain in general direction .
G_e	=	Gain of element of integration .
G_m	=	Particular value of G .
G_n	=	Particular value of G .
G_o	=	Antenna gain relative to uniform illumination case .
G_1	=	Gain of antenna number 1 .
G_2	=	Gain of antenna number 2 .
g	=	Pre-amplifier gain .
k	=	Boltzmann's Constant .
L	=	Radial distance to point within antenna aperture .
L'	=	AL
L_o	=	Radius of complete antenna .
m	=	Summation integer .
N	=	Noise power from complete antenna (watts).
N_o	=	Noise power output from directional coupler.
N_1	=	Noise power from antenna number 1 .
N_2	=	Noise power from antenna number 2 .
n	=	Summation integer .
R	=	b/a
r	=	Normalized radius of reflector .
S_o	=	Signal power output from directional coupler .
S_1	=	Signal power from antenna number 1 .
S_2	=	Signal power from antenna number 2 .
$\left(\frac{S}{N}\right)$	=	General Symbol for signal-to-noise ratio .
T	=	Ray temperature in general direction .
T_A	=	Temperature of lossy antenna .
T_a	=	Temperature of lossless antenna .
$T_{A'}$	=	Temperature of antenna pointed at sun .
T_c	=	Cosmic noise temperature .
T_e	=	Temperature of element of integration .
T_F	=	Pre-amplifier noise temperature .
T_G	=	Ambient temperature of ground, air and antenna .

T_m = Particular value of T .
T_n = Particular value of T .
T_s = Sky temperature .
T_o = Temperature of antenna elements .
T_1 = Temperature of antenna number 1 .
T_2 = Temperature of antenna number 2 .
u = S_1/N_1
v = S_2/N_2
w = N_2/N_1
x = Angle from axis of antenna beam (degrees).
α = Transmission coefficient of atmosphere .
β = Transmission coefficient of antenna feed system .
Γ = Gain-temperature ratio relative to isotropic radiator at 1^{o}K .
Γ_e = Gain-temperature ratio of element of integration .
Γ_o = Gain-temperature ratio relative to ideal antenna .
θ = Angle of general ray to antenna axis .
$|\rho|^2$ = Power reflection coefficient of ground .
$d\Omega$ = Element of solid angle .

TABLE OF CONTENTS

SOME PRINCIPLES OF LOW NOISE ANTENNA DESIGN

I. INTRODUCTION.

In the past the major sources of noise in any receiving system were the initial amplifying and mixing stages. With the advent of parametric amplifiers and masers, however, the emphasis has shifted to the antenna. In many applications, for example tropospheric scatter reception, the antenna is the major source of noise to the extent that further improvement in receiver design would yield little or no benefit. In other applications, for example satellite communications, the receiver may still contribute the major portion of the noise provided the antenna has been designed with low noise in mind. In this report the principal sources of antenna noise will be outlined. A number of basic principles of low noise antenna design will then be discussed and a series of criteria derived for the design and evaluation of such antennas.

II. ANTENNA TEMPERATURE.

In studying antenna noise it is convenient to use the concept of antenna temperature. The antenna temperature is defined as the physical temperature (measured in degrees Kelvin) of the matched load which, if substituted for the antenna, would produce the same noise output from the receiver. It will be apparent that the matched load referred to in this case is the equivalent to the "black body" of thermodynamics. Antenna noise is thus thermal in origin, being a combination of thermal energy emanating from lossy portions of the antenna structure, the ground, the atmosphere and cosmic sources. The latter include the sun and radio stars. It may be argued that some cosmic noise is not of thermal origin, However, this is not important to the antenna designer and noise temperature is still a very useful means of measuring this radiation. The noise power available at the antenna terminals is related to the antenna temperature by the following, well known equation.

$$N = k \cdot T_A \cdot B \tag{1}$$

where N is the noise power in watts

k is Boltzmann's constant (1.38×10^{-23} joules/degree Kelvin)

T_A is the antenna temperature in degrees Kelvin

B is the bandwidth in cycles per second

The noise power thus has a linear relationship to the antenna temperature. However, when noise power is used a specific bandwidth is implied. The use of noise temperature implies no such restriction

and is therefore, in general, a more useful quantity when evaluating antenna systems.

In evaluating the antenna temperature the case of a lossless antenna will be considered first. The term lossless is used here to mean that there are no resistive losses within the antenna structure itself. The antenna temperature is then given by the equation:

$$T_a = \frac{1}{4\pi} \int G \cdot T \cdot d\Omega \tag{2}$$

where: T_a = temperature of lossless antenna
$d\Omega$ = element of solid angle
T = temperature of radiation from $d\Omega$
G = antenna gain in direction of $d\Omega$
and the integration is performed over the whole sphere and for both polarisations. In practice, of course, G is usually zero for one polarisation, at least within the main beam. However, it may sometimes be necessary to take both polarisation into account when considering the back of the antenna. To evaluate Eq. (2) it is, therefore, necessary to know the complete radiation pattern of the antenna and the ray temperature T, over the whole sphere. Each must be known for both polarisations.

The numerical evaluation of Eq. (2) is usually simplified by the fact that the integration can often be broken down into a few broad regions in each of which either G or T may be regarded as constant. The integration then reduces to finding an average value for the remaining variable in each region. Eq. (2) may then be rewritten:

$$T_a = \frac{1}{4\pi} \left[\sum_n G_n \cdot \int T \cdot d\Omega \quad + \sum_m T_m \int G \cdot d\Omega \right] \tag{3}$$

where n and m are both small integers and the integrations are performed over specific regions of the sphere. As an example, consider the case of an antenna having a narrow main beam with the rest of the minor lobes distributed evenly over the sphere in a situation where the ray temperature is constant over the main beam and the gain may be regarded as having a constant average value in the minor lobe region. Eq. (3) then reduces to two terms with n and m both equal to one.

III. SOURCES OF NOISE RADIATION.

Although many practical low noise antenna problems may be simplified in the manner just described, it is in general necessary to know the antenna radiation pattern and the noise temperature distribution over the complete sphere. The antenna radiation pattern will

not be discussed in detail here. It suffices to say that the whole pattern is important, not just the main beam and the first one or two side lobes. It is also necessary to consider both polarisations in the regions where an appreciable cross polarised component exists.

For an antenna operating at a few hundred megacycles or higher, the principal sources of noise radiation are the cosmic noise, and thermal radiation from the atmosphere and the ground. The cosmic noise received is a function of the direction of the antenna beam with respect to the galactic center and also of frequency. It decreases with increasing frequency until it becomes negligible in the microwave band, with the exception of the sun and possibly a few radio stars if they are within the main beam.[1,2] The effect of the atmosphere may be represented for practical purposes by a simplified model in which the actual atmosphere is replaced by an equivalent uniform atmosphere, at normal pressure, 8 km thick, and refraction effects are accounted for by assuming that the effective radius of the earth is 4/3 the actual radius. Attenuation occurs as a result of oxygen and water vapour absorption. The attenuation of a ray passing through the atmosphere will thus be a function of elevation angle and may be readily calculated from the geometry of the uniform equivalent atmosphere.[3,4] Thus the temperature of a ray incident on the antenna from above the horizon will be given by Eq. (4)

$$T = \alpha \cdot T_c + (1 - \alpha)\, T_G \tag{4}$$

where: T = ray temperature
α = transmission coefficient of atmosphere in direction of the ray
T_c = cosmic background temperature in direction of the ray
T_G = ambient temperature of atmosphere

The final contribution to noise radiation in the vicinity of the antenna is that of the thermal radiation emitted by the ground. In general this requires a more complex treatment. Some surfaces , such as a rough sea or dense vegetation, which are "rough" in terms of a wavelength, absorb almost all radiation incident on them and must be treated as black bodies. Other surfaces such as concrete or short grass, which are "smooth" in terms of a wavelength, behave as dielectric interfaces and reflect a greater or lesser portion of the incident radiation depending on the incident angle, the dielectric constant, and the plane of polarisation.[5] The effect of this reflection on the noise temperature may be calculated in the same way as the effect of atmospheric attenuation, the transmission coefficient now being the power reflection coefficient of the ground. The temperature of a ray incident on the antenna from below the horizon is then given by Eq. (5)

$$(5) \qquad T = |\rho|^2 \left[\alpha \cdot T_c + (1-\alpha) T_G \right] + \left[1 - |\rho|^2 \right] T_G$$

where $|\rho|^2$ is the power reflection coefficient of the ground. It is assumed that the ambient temperature of the ground and of the atmosphere are the same; an assumption which in practice is justified.

The ray temperature may now be calculated in all directions round the antenna using Eqs. (4) and (5) . The results may then be plotted to yield a noise map of the environment of the antenna. Temperatures calculated in this manner have been found to be in good agreement with those obtained experimentally with a radiometer. Some typical measured temperatures, obtained with the X-band radiometer described later in this report, are shown in Fig. 1 . These curves have been corrected for feedline loss only and, therefore, include any effects of side lobes, beamwidth, spillover and so on. It will be observed that the grass behaves as a rough surface, the temperature rising sharply at the horizon and being largely independant of polarisation. The ashphalt, however, behaves as a smooth surface and yields appreciably lower temperatures, particularly for horizontal polarisation and shallow incidence angles. The sharp spike near the horizon, caused by the presence of houses and a ploughed field, serves to emphasise the important influence of environment on antenna temperature. Further details of antenna noise temperature calculations may be found in reference 6 .

So far only lossless antennas have been considered. Loss may occur as a result of various factors such as attenuation in feedlines or lenses or at reflecting surfaces. Mismatch loss may also be treated in the same manner if it is assumed that some isolating device at ambient temperature, such as circulator, is placed between the antenna and the receiver. The effect of such loss is similar to that of attenuation in the atmosphere and the noise temperature of a lossy antenna is thus given by Eq. (6)

$$(6) \qquad T_A = \beta \cdot T_a + (1-\beta)\, T_G$$

where: T_A = temperature of lossy antenna
T_a = temperature of equivalent lossless antenna
β = transmission coefficient of the lossy portions of the antenna structure
T_G = ambient temperature

The ambient temperature is usually assumed to be the same for the atmosphere, the ground and the antenna, unless special precautions are taken to cool the lossy portions of the antenna. The effect of such losses is shown in Fig. 2 .

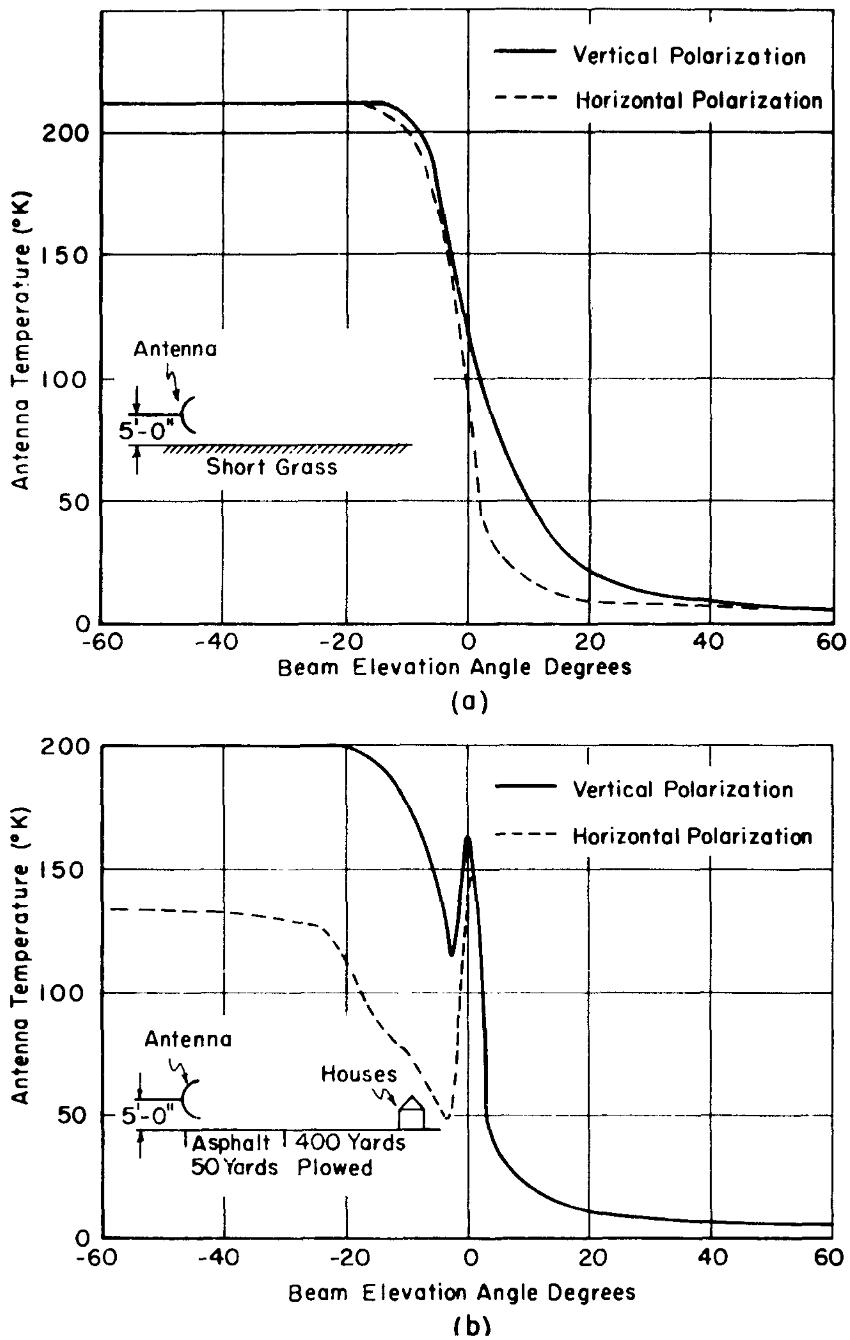

Fig. 1. Measured temperatures of a 4 ft paraboloidal antenna at X-band as a function of beam elevation angle.

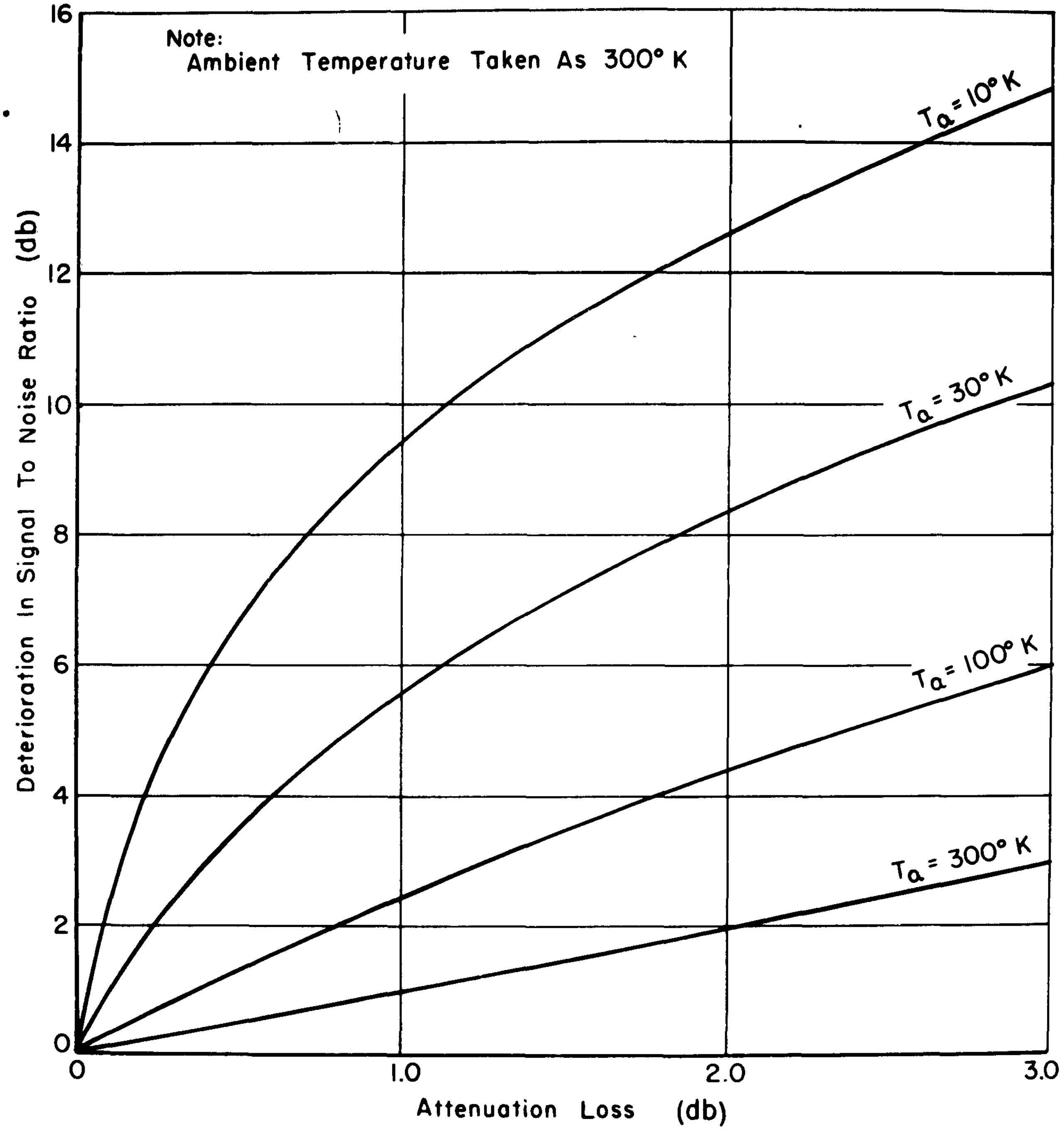

Fig. 2. Graph showing the effect of attenuation loss on the signal to noise ratio of a low noise antenna in terms of the noise temperature (T_a) of the equivalent lossless antenna.

IV. ANTENNA GAIN-TEMPERATURE RATIO.

In any receiving system the ultimate measure of system performance is the signal-to-noise ratio at the output of the receiver. When a low noise receiver is used this mainly depends on the signal-to-noise ratio at the output of the antenna. It is, therefore, important to consider antenna noise not as a separate entity but rather to use signal-to-noise ratio as the design criterion, since any change in the antenna design will, in general, affect both the signal and the noise levels.

The signal and noise powers are not in themselves convenient quantities to use in describing the performance of a receiving antenna since they both depend on factors unrelated to the antenna. The signal power depends on the transmiting system and the intervening medium. Gain is thus a more convenient quantity to represent the effect of the receiving antenna on signal power. Noise power, as already mentioned, depends on receiver bandwidth. Noise temperature is, therefore, a more convenient quantity to use here. The performance characteristic of a low noise antenna may thus be expressed in terms of its gain to noise temperature ratio. A convenient reference point for gain is the isotropic radiator and for noise a temperature of $1^{o}K$. The performance of a low noise antenna may thus be conveniently expressed in terms of its Gain-Temperature ratio measured relative to an isotropic radiator at one degree Kelvin. This ratio will be denoted by Γ in this report. It is often convenient to express Γ in decibels. For example an antenna having a gain of 30 db and a noise temperature of $100^{o}K$ would have a gain-temperature ratio of +10 db on such a scale of reference. However, the same antenna looking in a different direction might have a noise temperature of only $10^{o}K$. The gain-temperature ratio would be +20db. The gain-temperature ratio is thus fixed only for a particular antenna having a specific orientation in a given environment. This serves to emphasise the importance of designing low noise antennas for a particular application. An antenna which performs well for one orientation may be poor for another.

V. A HYPOTHETICAL "IDEAL ANTENNA".

The gain-temperature ratio discussed above provides a measure of the absolute performance of a low noise antenna in a given situation. However, it does not provide the designer with any indication of whether this performance is good or bad for the size of aperture employed. To fill this need it has been found convenient to define a hypothetical "Ideal Antenna" as follows:

> The "Ideal Antenna" is defined as a lossless antenna having the gain of a uniformly illuminated aperture, equal in area to that of the antenna

with which it is compared, and a noise temperature equal to the ray temperature in the direction of the signal source.

This "Ideal Antenna" has the maximum possible gain and also the minimum possible noise temperature for a given situation, since in effect the definition assumes that the antenna radiation pattern is confined to a single narrow beam and hence has no side or back lobes to pick up stray thermal radiation. There are possible exceptions, one being if an antenna having a truly uniformly illuminated aperture is orientated in such a way that all the minor lobes are at a lower temperature than the main beam. There would then be a slight improvement over the above definition. However, the difference would be marginal and, in practice, it is virtually impossible for such a situation to occur. Another exception is when an intense discrete noise source is directly in line with the signal source. The definition would then yield an artificially low value. However, such freak situations excepted, the "Ideal Antenna" gives the maximum possible signal-to-noise ratio in any given case and the performance of any practical antenna may be described in terms of the amount by which its gain-temperature ratio falls below that of the "Ideal". The gain-temperature ratio relative to that of the "Ideal Antenna" will be denoted by Γ_0 .

VI. AN EXAMPLE OF A LOW NOISE ANTENNA DESIGN.

An example of a low noise antenna design problem will now be given to illustrate the importance of designing for maximum gain-temperature ratio, rather than for maximum gain alone . The case to be considered is that of a parabaloid operating in the microwave band and used for receiving signals from an artificial satellite. The antenna is operated with the main beam pointing 20° or more above the horizon and it will be assumed that the sky temperature is a constant 10°K in this region. The antenna is located over ground which is rough in terms of a wavelength and so may be regarded as having a uniform noise temperature of 300°K. The parabaloid will be assumed to have a large focal length to diameter ratio in order to simplify the geometry for the purpose of this illustration. The basic approach would of course be the same in all cases. This assumption means that the angle subtended by the reflector at the focus is small enough for the small angle approximations of trigonometry to apply. It also means that any "spill-over" energy entering the feed around the edges of the reflector will be from the ground. It will also be assumed that the radiation pattern of the feed has circular symmetry and a cosine squared shape with no side or back lobes. The situation is illustrated in Fig. 3 . The efficiency will be calculated first. This is the fraction of the total feed power which is incident on the reflector and is given by Eqs. (7) and (8).

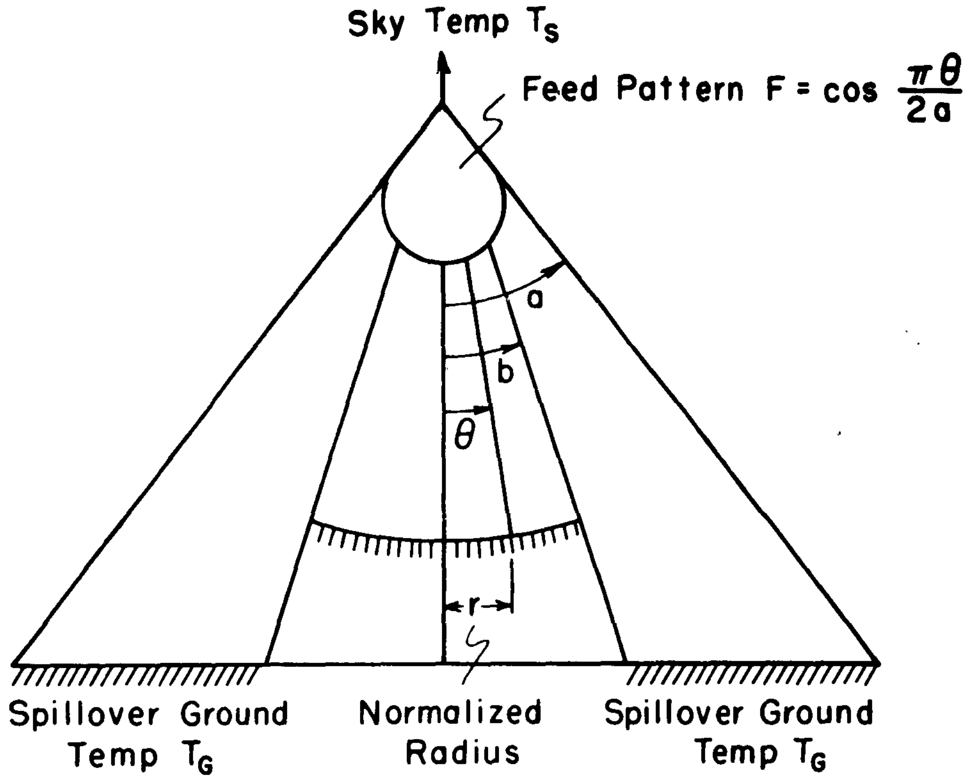

Fig. 3. The feed geometry of a long focal length parabaloid .

(7) $$E = \frac{\int_0^b 2\pi\theta \cos^2 \frac{\pi\theta}{2a} \cdot d\theta}{\int_0^a 2\pi\theta \cos^2 \frac{\pi\theta}{2a} \, d\theta}$$

(8) $$\therefore E = \frac{\pi^2 R^2 + 2\cos \pi R + 2\pi R \cdot \sin \pi R - 2}{\pi^2 - 4}$$

where $R = \frac{b}{a}$

The antenna noise temperature may be calculated directly from the efficiency on the assumption that all power incident on the reflector is at the sky temperature (T_S) and all spillover power is at the ground temperature (T_G) .

(9) $$T_a = E \cdot T_s + (1 - E)\, T_G \; .$$

The next stage is to calculate the directivity of the antenna. A relative value for this in terms of the normalised radius of the reflector (r) is given by Eq. (10).

(10)
$$D = \frac{\left[\int_0^1 \cos\left(\frac{\pi b r}{2a}\right) . 2 \pi r \, d r \right]^2}{\int_0^1 \cos^2 \left(\frac{\pi b r}{2a}\right) . 2 \pi r \, d r} .$$

Therefore,

(11)
$$D = \frac{128 \left[\cos \frac{R\pi}{2} + \frac{R\pi}{2} \sin \frac{R\pi}{2} - 1 \right]^2}{\pi R^2 \left[\pi^2 R^2 + 2\cos \pi R + 2 \pi R \sin \pi R - 2 \right]}$$

where R again equals b/a .

When R is very small the illumination will be uniform . Then by using the small angle approximation for sine and cosine and letting R tend to zero the limiting value of D for the uniform illumination case may be found. If this is denoted by D' then from Eq. (11) :

(12)
$$D' = \pi$$

The directivity for any other illumination, in terms of the uniform illumination case, may then be expressed by D_o where :

(13)
$$D_o = \frac{D}{D'}$$

Therefore,

(14)
$$D_o = \frac{128 \left[\cos \frac{R\pi}{2} + \frac{R\pi}{2} \sin \frac{R\pi}{2} - 1 \right]^2}{\pi^2 R^2 \left[\pi^2 R^2 + 2\cos \pi R + 2\pi R \sin \pi R - 2 \right]}$$

The gain relative to a uniformly illuminated aperture is then given by :

(15)
$$G_o = D_o E$$

Combining Eqs. (8) and (15) : -

(16)
$$G_o = \frac{128}{\pi^2 R^2 (\pi^2 - 4)} \left[\cos \frac{R\pi}{2} + \frac{R\pi}{2} \sin \frac{R\pi}{2} - 1 \right]^2 .$$

Eq. (16) gives the gain relative to a uniformly illuminated aperture with 100% efficiency. The gain-temperature ratio relative to the "Ideal Antenna" is given by :

$$\Gamma_o = \frac{G_o \cdot T_s}{T_a} \tag{17}$$

Combining Eqs. (9) and (17) :

$$\Gamma_o = \frac{G_o \cdot T_s}{T_G - E\,(T_G - T_s)} \tag{18}$$

where E is obtained from Eq. (8) . G_o and Γ_o may now be plotted as functions of R . This has been done in Fig. 4 for the case of $T_G = 300^oK$ and $T_s = 10^oK$. The third curve in the figure shows the intensity of illumination at the edge of the reflector compared to that at the center. It will be observed that maximum gain occurs when R = 0.8 or when the illumination is 10 db down at the edge of the reflector. However, maximum gain-temperature ratio, and hence maximum signal-to-noise ratio occurs when R is about 0.94, or an illumination taper of about 16 to 20 db. The gain is down by about 0.5 db . However, had a maximum gain antenna been used in this case, the signal-to-noise ratio would have been nearly 3 db below the maximum. Designing for maximum gain-temperature ratio is, therefore, of the greatest importance . A detailed study of the parabaloid for use as a low noise antenna is contained in Ref. 7 .

VII. VERY HIGH GAIN ANTENNAS.

So far the discussion has dealt with the problem of maximising the signal-to-noise ratio for an antenna of given aperture. The limit is reached when the gain-temperature ratio approaches that given by the Ideal Antenna definition. If a further increase in signal-to-noise ratio is still required the only recourse is to increase the aperture of the antenna. However, even this has its limitations because of the practical problems introduced when very large apertures are used. The first of these problems is that of mechanical tollerances. It may prove impossible, or at least very costly, to construct a very large aperture to a sufficiently high degree of accuracy and to maintain this accuracy under operating conditions. If this is not insurmountable then there arises the question of the phase coherence of the received wavefront across the aperture. Due to various meteorological conditions prevailing along the transmission path it is found that the phase of a received wave is generally not constant when examined over a wide front. The significance of this is illustrated in Fig. 5 . A wavefront is shown,

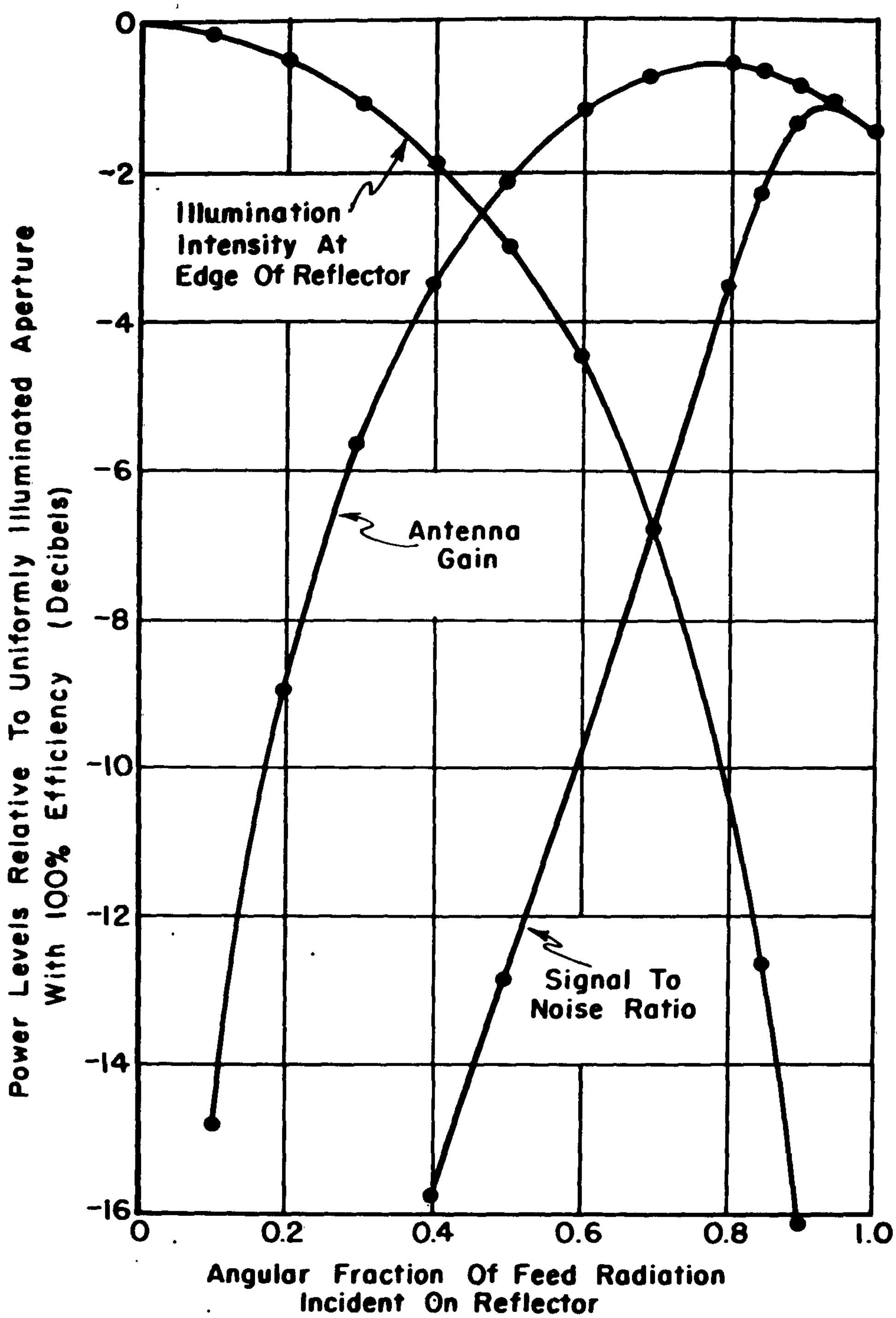

Fig. 4. The optimum illumination for a long focal length paraboloid used as a low noise antenna and looking vertically upwards.

incident on various apertures and for which the phase varies with the position across the front. For an antenna with aperture AA the phase is substantially constant over the aperture and no problem will be encountered. Across the aperture BB the wave is still substantially plane but exhibits a linear phase error from one side to the other. This antenna will thus still observe a point source of signal but one which has a tendency to move around, sometimes moving out of the antenna beam. The antenna BB would thus be satisfactory if provision were made for tracking this apparent movement of the signal source

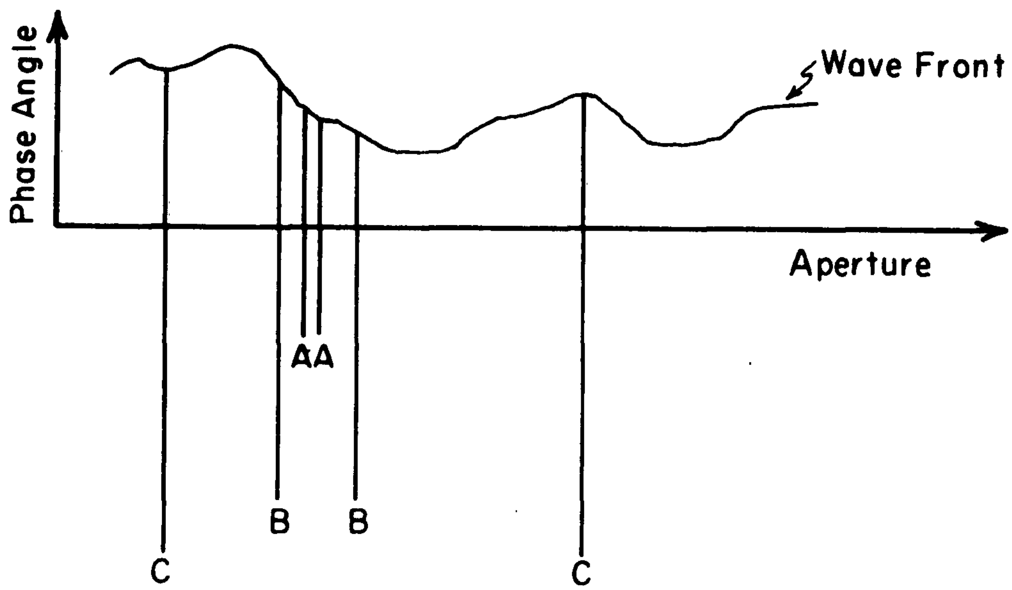

Fig 5 The effect of a distorted wavefront on large aperture antennas

For an even wideraperture CC the phase error would be largely random This antenna would thus observe a general blurring of the signal source with consequent loss of gain. There would be no point in using an aperture of this size in practice. The physical size of the apertures falling into these three categories will depend on the path conditions in each case The division between one category and another will be for a smaller aperture in the case of scatter type reception than for example a line of sight link between a ground receiver and a space probe Waterman[8] has observed than the division between categories AA and BB occurs for beamwidths around 1 or 2 degrees for tropospheric scatter links. An experiment to determine the size limitations of X band antennas used for ground to satellite or ground to space probe communications is now being conducted at this laboratory The experiment consists of observing the apparent position of the sun with an X-band radiometer and is described in more detail later in this report

VIII. MULTI ELEMENT ANTENNA.

From the previous section it will be seen that the useful size of receiving antennas with continuous apertures is limited to the first two categories. However, if still larger apertures are required the problem may be overcome by the use of multielement arrays. These would not be arrays of small elements, such as dipoles or slots, but rather arrays of elements which are themselves high gain antennas with gains of 30 db or more. The only restriction on element size would be that it came within the first of the three categories previously described. Each of these elements would then be coupled to the receiver via an individual phase correcting servo system. It would be necessary to restrict the banwidth of the receivers used in the phase correction system in order to obtain a sufficiently high signal-to-noise ratio from each element individually. However, since changes in the phase front are the result of physical motion of the atmosphere they would occur at relatively low frequencies. This bandwidth restriction would thus be no problem. The maximum possible bandwidth would of course be used for the main receiver to which each antenna element would be connected in phase. The feasability of such a system has been demonstrated using two antenna elements, the experiment being described in section XV of this report.

IX. DETERMINING THE SIGNAL-TO-NOISE RATIO OF A MULTI-ELEMENT ANTENNA SYSTEM.

A multielement system of the type just described can be made to yield an increase in gain. However, at the same time additional attenuation loss, with its attendant noise, has been introduced by the addition of phase shifters and interconnecting feedlines. It is, therefore, necessary to determine whether the gain-temperature ratio can be improved by the use of this multielement system and if so to maximise it. It will be immediately evident that much tedious computation is involved when the signal and noise powers from a large number of antenna elements must be combined in a lossy network of feedlines and the whole system optimised to yield the best possible performance. A mathematical method of handling signal-to-noise ratio as a single parameter, instead of handling the signal and noise powers separately, has therefore been developed. The method applies to gain-temperature ratio as well as to signal-to-noise ratio since the two are linearly related.

X. COMBINING TWO SIGNAL-TO-NOISE RATIOS.

Consider two receiving antennas, or other signal sources, connected to a receiver via a lossless coupler as shown in Fig. 6. The coupler is so arranged that the two signals will be in phase in the receiver

arm and has a power coupling factor c in the sense indicated in the figure. The signals will be phase coherent and must therefore be treated as vectors. The noise powers , on the other hand, will generally

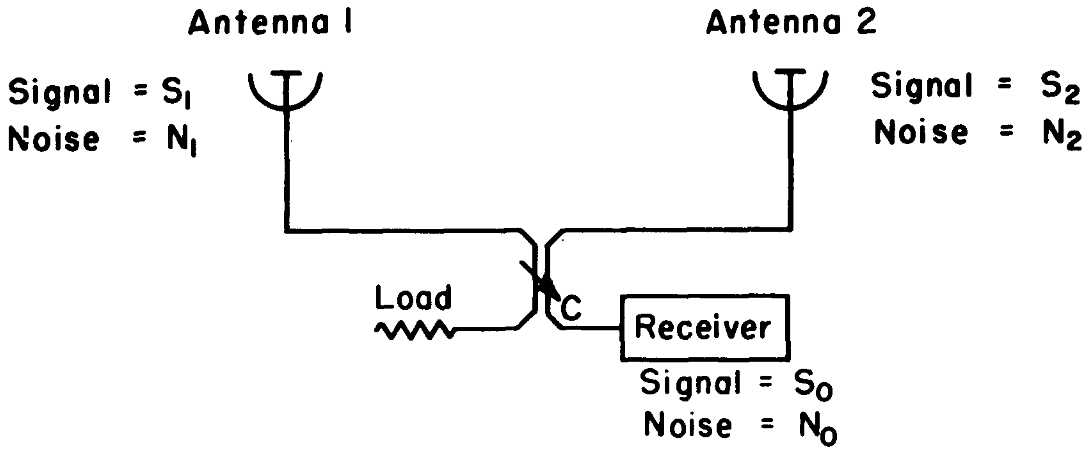

Fig. 6. Combining two signal-to-noise ratios.

be incoherent and must be regarded as scalars. This latter statement requires some qualification. Noise power originating from different sources, such as different sections of lossy feedline, will be incoherent, but in the case of two antennas looking at the same region of space, a portion of the noise originates from the same group of sources and will be phase coherent. Thus, when two antennas are used, the system will exhibit an increase in gain with respect to these noise sources. At the same time there will be a corresponding reduction in beamwidth with the result that, if the noise sources are evenly distributed over the region of space common to the two individual antenna beams, the total noise obtained when the two antenna outputs are combined will be the same as if it originated from entirely separate sources. However, if the noise sources are not evenly distributed within the antenna beams, as for example if a discrete source is located in this region, then the sources must be treated individually and the antenna outputs added with appropriate allowance for phase. In practice a single discrete source for example could probably be treated as an isolated point superimposed on a uniform background of noise. The following argument, therefore, applies to the case when the noise within the antenna beams is evenly distributed. The noise outputs from the two antennas will , therefore, be regarded as completely incoherent. This is obviously

valid for such portions of the noise as originate from separate sources and yields the correct result for the portions originating from the same sources for the reasons just discussed.

Let S_1 be the signal power from antenna 1
Let S_2 be the signal power from antenna 2
Let N_1 be the noise power from antenna 1
Let N_2 be the noise power from antenna 2
Let S_o be the signal power entering the receiver
Let N_o be the noise power entering the receiver
Let c be the power coupling factor (see Fig. 6).

Also let.

$$\frac{S_1}{N_1} = u \qquad (19)$$

$$\frac{S_2}{N_2} = v$$

$$\frac{N_2}{N_1} = w$$

Now the signal power entering the receiver will be given by

$$S_o = \left[\sqrt{c\, S_1} + \sqrt{(1-c)\, S_2} \right]^2 \qquad (20)$$

and the noise power by

$$N_o = c\, N_1 + (1-c)\, N_2 \; . \qquad (21)$$

Combining Eqs. (19), (20) and (21) yields

$$\frac{S_o}{N_o} = \frac{\left[\sqrt{c\, u} + \sqrt{(1-c)\, v\, w} \right]^2}{c + (1-c)\, w} \qquad (22)$$

If the coupling factor c is now adjusted to yield the maximum signal-to-noise ratio then it follows that

$$\frac{d}{dc} \left(\frac{S_o}{N_o} \right) = 0 \; . \qquad (23)$$

Carrying out the differentiation and solving for c and (1-c) yields the results

$$c = \frac{u\, w}{u\, w + v} \; . \qquad (24)$$

(25) $$1 - c = \frac{v}{uw + v} \quad .$$

Substituting these results in Eq. (22) gives:

(26) $$\left(\frac{S_o}{N_o}\right)_{max} = \frac{\left[\sqrt{u^2 w} + \sqrt{v^2 w}\right]^2}{uw + vw} ,$$

(27) $$\left(\frac{S_o}{N_o}\right)_{max} = u + v \; .$$

Thus if the coupling factor is optimised the combined signal-to-noise ratio is simply the sum of the two input ratios. This result may be extended by mathematical induction to any number of antennas and also to integration methods when these are appropriate.

This method will very quickly yield a figure for the maximum signal-to-noise ratio which may be obtained from a given antenna system. In order to design a practical antenna it is, however, necessary to evaluate the optimum coupling factor at each junction . Substituting the signal and noise powers back into Eq. (24) yields :

(28) $$c = \frac{N_2^2 \; S_1}{N_2^2 \; S_1 + N_1^2 \; S_2} \quad .$$

Since both noise and signal appear as terms of the same power in both numerator and denominator a direct substitution of gain and noise temperature may be made for the signal and noise powers respectively, provided the bandwidth between the various parts of the antenna and receiving systems is compatible . Thus Eq. (28) becomes :

(29) $$c = \frac{T_2^2 \; G_1}{T_2^2 \; G_1 + T_1^2 \; G_2} \quad .$$

XI. LIMITATIONS ON THE PERFORMANCE OF VERY LARGE ARRAYS .

Since the problems of mechanical tolerances and random errors across the wavefront may be overcome by the use of multielement arrays with servo driven phase shifters, the question may well be asked as to whether there is any limit to the size of such an array

or to the signal-to-noise ratio which can be obtained from it. The answer is that there is a limit determined by the quality of the feed-lines which interconnect the array. To illustrate this two examples will be considered. In both cases it will be assumed that the arrays are extremely large to the extent that the relatively large elements of which they consist are still small in terms of the total aperture. The normal methods of mathematical integration may, therefore, be used as if the aperture was continuous.

XII. A CIRCULAR ARRAY WITH MINIMUM LENGTH FEEDLINES.

It is selfevident that the array with least feedlineloss will be a circular array with the receiver located at the center and connected to each element by the shortest possible length of feedline. If each element is uniformly illuminated then the gain of an annular ring of elements (Fig. 7), as seen from the receiver is given by

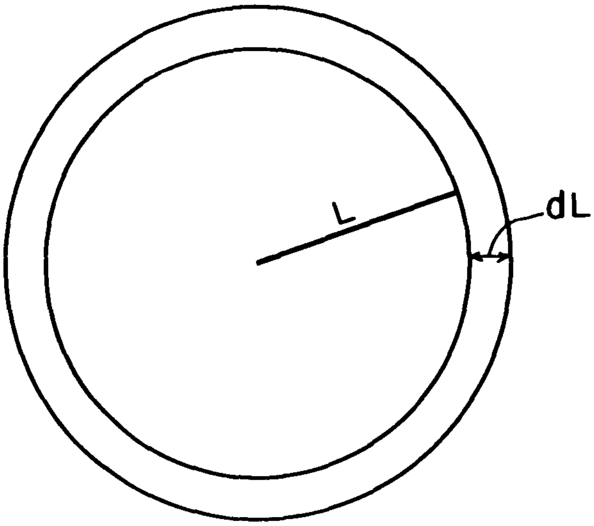

Fig. 7. An element of a minimum loss array .

$$G_e = \frac{4\pi}{\lambda^2} \cdot e^{-AL} \cdot 2\pi \, L \, d\,L \tag{30}$$

where G_e is the gain of the element
L is the radius of the element
e^{-AL} is the transmission coefficient of the feedline .

The noise temperature of the element will be given by

$$T_e = e^{-AL}\, T_\Theta + (1 - e^{-AL})\, T_G \tag{31}$$

where T_e is the temperature of the element. Combining these, the gain-temperature ratio, as seen from the receiver is

$$(32)\qquad \Gamma_e = \frac{8\pi^2}{\lambda^2} \cdot \frac{L\, e^{-AL}\, dL}{e^{-AL}\, T_o + (1 - e^{-AL})\, T_G} \ .$$

If optimum coupling is now used the signal-to-noise ratio, and hence the gain-temperature ratio, for the whole antenna will simply be the sum of the ratios for the individual elements. Thus the maximum gain-temperature ratio which may be obtained from the antenna is found by integrating Eq. (32). Thus

$$(33)\qquad \Gamma = \frac{8\pi^2}{\lambda^2 \cdot T_G} \int_o^{L_o} \frac{L\, dL}{e^{AL} - (1 - \frac{T_o}{T_G})}$$

where L_o is the radius of the complete antenna. The general solution of this integral becomes very involved. However, the maximum possible value for Γ may be readily calculated. This maximum value occurs when L_o is infinite and when T_o is so small that it can be neglected in comparison to T_G. Eq. (33) may then be written

$$(34)\qquad \Gamma_{max} = \frac{8\pi^2}{\lambda^2\, T_G} \int_o^{\infty} \frac{L\, dL}{e^{AL} - 1}$$

By making the substitution

$$(35)\qquad L' = AL$$

Eq. (34) becomes a standard integral.

$$(36)\qquad \Gamma_{max} = \frac{8\pi^2}{\lambda^2 \cdot T_G \cdot A^2} \int_o^{\infty} \frac{L'\, dL'}{e^{L'} - 1}$$

Evaluating the integral from tables[9] gives :

$$(37)\qquad \Gamma_{max} = \frac{8\pi^2}{\lambda^2 \cdot T_G \cdot A^2} \cdot \frac{\pi^2}{6} \ ,$$

therefore

$$(38)\qquad \Gamma_{max} = \frac{4\pi^2}{A^2\, \lambda^2\, T_G} \cdot \frac{\pi^2}{3} \ .$$

This is the maximum value of Γ which can be obtained from any array of this type. Although this result has been calculated for an infinite aperture, values closely approaching this could be obtained in practice from apertures of the order of a few thousand wavelengths.

For comparison the maximum gain obtainable from this type of array will now be calculated. Eq. (30) gives :

$$(39) \qquad G_e = \frac{4\pi}{\lambda^2} \cdot e^{-AL} \cdot 2\pi L \, dL .$$

If optimum coupling is used, maximum gain will be obtained from an infinite aperture thus :

$$(40) \qquad (G)_{max} = \frac{8\pi^2}{\lambda^2} \int_0^\infty L \, e^{-AL} \, dL ,$$

$$(41) \qquad (G)_{max} = \frac{8 \pi^2}{A^2 \lambda^2} .$$

Again this result is for an infinite aperture but in practice gains closely approaching this can be obtained from apertures of a few thousand wavelengths. It should also be remembered that the optimum coupling conditions for maximum gain and maximum signal-to-noise ratio are not necessarilly the same.

XIII. A CIRCULAR ARRAY WITH EQUAL LENGTH FEEDLINES.

The array just described, with minimum length feedlines, will yield the highest gain and signal-to-noise ratio. However, there will be large differences in the feedline length required to reach different parts of the array. Because of the necessity for maintaining the correct phase relationship throughout the array, this will result in a very narrow bandwidth. This problem may be overcome by using equal length feedlines to all parts of the antenna. The antenna will then be inherently broad banded, although some restriction may be encontered when the antenna is scanned by moving the individual elements on account of the different pathlengths produced outside the antenna. This type of antenna suffers from the disadvantage that the feedline length must be sufficient to reach the most distant point of the aperture, with a resultant increase in feedline loss. Its gain-temperature ratio will now be calculated for comparison with the previous case.

Since the loss is the same for all parts of the aperture the gain may be written down immediately .

$$(42) \qquad G = \frac{4 \pi}{\lambda^2} \; \pi L_o^2 \cdot e^{-AL_o}$$

where L_o = radius of aperture . Similarly the noise temperature is given by

$$(43) \qquad T_A = e^{-AL_o} \; T_o + (1 - e^{-AL_o}) \; T_G \, ,$$

Therefore

$$(44) \qquad \Gamma = \frac{4\pi^2}{\lambda^2 T_G} \cdot \frac{L_o^2}{e^{AL_o} - \left(1 - \frac{T_o}{T_G}\right)} \, .$$

The value of L_o which will give a maximum value of Γ may be found in the usual manner by differentiating Eq.(44) with respect to L_o and equating the derivative to zero. This yields the condition :

$$(45) \qquad e^{AL_o} \left(1 - \frac{AL_o}{2}\right) = 1 - \frac{T_o}{T_G} \quad .$$

This equation is trancendental but may be readily solved for specific numerical values. For comparison with the minimum loss case consider the situation when T_o is very small compared with T_G . Equation (45) then becomes :

$$(46) \qquad e^{AL_o}\left(1 - \frac{AL_o}{2}\right) = 1 \, .$$

A graphical solution yields the result :

$$(47) \qquad AL_o = 1.59 \quad .$$

The maximum value for Γ may then be found by substituting back into Eq. (44) and simplifying :

$$(48) \qquad (\Gamma)_{max} = \frac{4 \; \pi^2}{A^2 \; \lambda^2 \; T_G} \times 0.648 \quad .$$

It will be noticed that this is smaller than the value obtained in Eq.(38) by a factor of approximately five to one .

The maximum gain condition may be found in a similar fashion by differentiating Eq. (42) and equating to zero. Thus for a maximum:

$$(49) \qquad L_o = \frac{2}{A}$$

Substituting this back in Eq. (42) gives

$$(50) \qquad G_{max} = \frac{4 \; \pi}{\lambda^2} \; \cdot \pi \left(\frac{2}{A}\right)^2 e^{-2}$$

Therefore

$$G_{max} = \frac{16\pi^2}{A^2 \lambda^2} \cdot \frac{1}{e^2} \quad . \tag{51}$$

This is smaller than the value calculated for the minimum loss case in Eq. (41), but the difference is not so large as the difference in signal-to-noise ratio. For easy comparison, these results are shown in tabular form in Fig. 8. To give a clearer impression of the practical quantities involved, numerical values of the various expressions have been calculated for the case of $T_G = 300^\circ K$ and $A = 0.0007$ wavelengths^{-1}. The value for A is based on an estimated attenuation of 0.6 db per 100 feet in brass waveguide at 2 KMc/s and is representative over the lower microwave frequencies. The numerical results in Fig. 8 are expressed in decibels, above an isotropic radiator in the case of gain and above an isotropic radiator at one degree Kelvin in the case of gain-temperature ratio.

XIV. PRE-AMPLIFIERS WITHIN AN ANTENNA ARRAY.

Since the principal factor limiting the gain-temperature ratio of large arrays is feedline loss, it is worthwhile to consider the possibility of using pre-amplifiers as a means of overcoming this. It must be remembered that an amplifier can never improve the signal-to-noise ratio since it will amplify both signal and noise equally. In addition it will add some noise of its own with the result that the output signal-to-noise ratio is always worse than that at the input. However, if the preamplifier must be connected to the main receiver by a long lossy cable then an improvement in the overall system may occur. The improvement results from the fact that the amplifier raises both the signal and the noise power to such a level that the cable no longer contributes a significant amount of noise. Consider the situation shown in Fig. 9. An antenna element is connected via a lossy transmission line to a receiver. At the terminals of the element there exist a signal power S_o and a noise temperature T_o. The transmission coefficient of the line is e^{-AL}. The signal-to-noise ratio at the receiver will then be:

$$\left(\frac{S}{N}\right)_1 = \frac{S_o e^{-AL}}{e^{-AL}\, T_o + (1 - e^{-AL})\, T_G} \quad . \tag{52}$$

If an amplifier having gain g and an equivalent noise temperature at its input of T_F is now inserted as in Fig. 9 the new signal-to-noise ratio will be given by :

$$\left(\frac{S}{N}\right)_2 = \frac{S_o g\, e^{-AL}}{e^{-AL} g\,(T_o + T_F) + (1 - e^{-AL})\, T_G} \quad . \tag{53}$$

		General Expression	Maximum Value	Numerical Maximum $\lambda A = 0.0007$ $T_G = 300^\circ$ K
Minimum length feedlines	GAIN "G"	$\frac{8\pi^2}{A^2\lambda^2}\left[1-e^{-AL_o}(1+AL_o)\right]$	$\frac{8\pi^2}{A^2\lambda^2}$	82 db
	Radius	L_o	∞	∞
	$\frac{\text{Gain}}{\text{Temperature}}$ "Γ"	$\frac{8\pi^2}{\lambda^2 T_G}\int_0^{L_o}\frac{L\,dL}{e^{AL}-\left(1-\frac{T_o}{T_G}\right)}$	$\frac{4\pi^2}{A^2\lambda^2 T_G}\cdot\frac{\pi^2}{3}$	59 db
	Radius	L_o	∞	∞
Equal length feedlines	GAIN "G"	$\frac{4\pi^2}{\lambda^2}\cdot L_o^2\, e^{-AL_o}$	$\frac{16\pi^2}{A^2\lambda^2}\,\frac{1}{e^2}$	76 db
	Radius	L_o	$\frac{2}{A}$	2860 wavelengths
	$\frac{\text{Gain}}{\text{Temperature}}$ "Γ"	$\frac{4\pi^2}{\lambda^2 T_G}\,\frac{L_o^2}{e^{AL_o}-\left(1-\frac{T_o}{T_G}\right)}$	$\frac{4\pi^2}{A^2\lambda^2 T_G}\times 0.648$	52 db
	Radius	L_o	$\frac{1.59}{A}$	2270 wavelengths

Fig. 8. Table of Values of Gain and Gain-Temperature Ratio for a Lossy Circular Array Fed from the center.

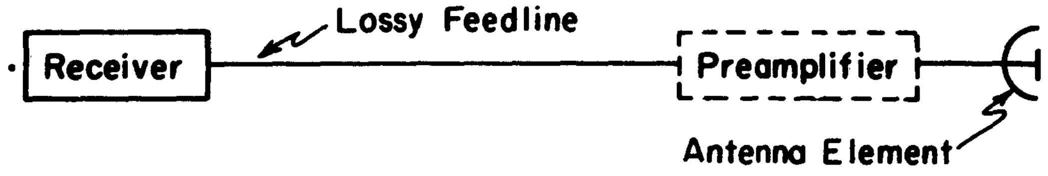

Fig. 9. THE USE OF A PRE-AMPLIFIER TO REDUCE THE EFFECT OF FEEDLINE LOSS

For the amplifier to yield an improvement $\left(\frac{S}{N}\right)_2$ must be greater than $\left(\frac{S}{N}\right)_1$ so from Eqs. (52) and (53)

$$T_F < \frac{1 - e^{-AL}}{e^{-AL}} T_G \cdot \left(1 - \frac{1}{g}\right) \tag{54}$$

or since the gain will generally be much greater than unity :

$$T_F < \left(\frac{1 - e^{-AL}}{e^{-AL}}\right) T_G \quad . \tag{55}$$

If the gain of the amplifier is sufficiently large both the signal and the noise powers at its output will be much larger than any noise contributed by the feedline. The signal-to-noise ratio at the far end of the feedline will then be substantially the same as that at the output of the amplifier. However, it must be remembered that since the feedline will attenuate both the signal and the noise from the amplifier, the gain must be sufficient to ensure that the above condition is valid throughout the length of the feedline.

XV. AN EXPERIMENTAL TWO ELEMENT ANTENNA.

In order to investigate the feasibility of multielement antenna systems a two-element model was constructed for radiation pattern studies. The system used is shown in schematic form in Fig. 10 . Two identical parabaloidal antenna were used, operating in Ku-Band. Each was 24 inches in diameter and fed by a waveguide in the shape of a shepherd's crook and terminated by a small horn. The two antennas were connected to the two side arms of a magic-T ; one via a servo-

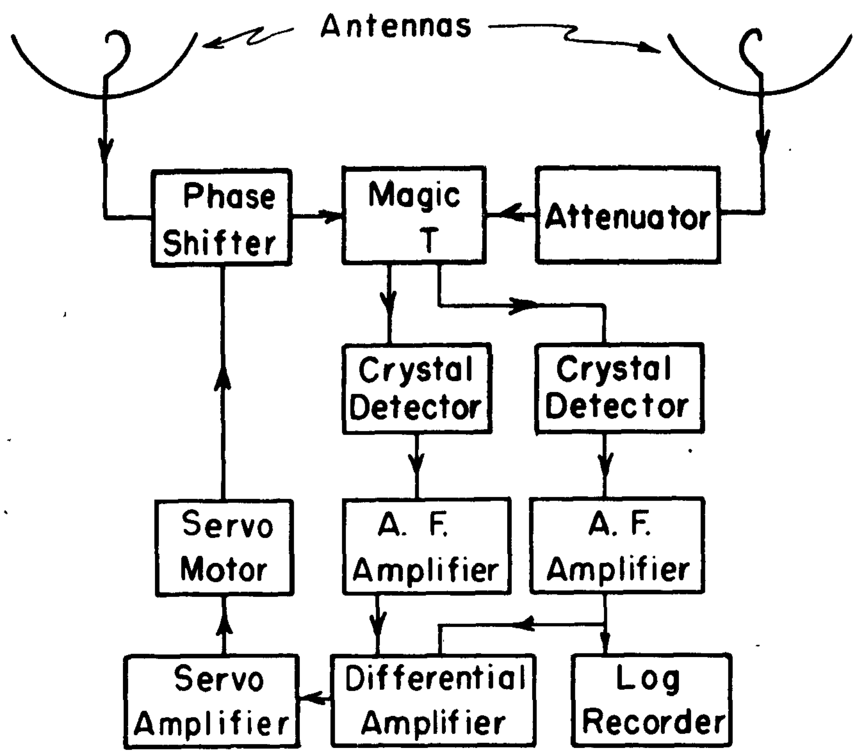

Fig. 10. Schematic diagram of an experimental two element phase correcting antenna system .

driver, rotary vane phase shifter; the other via a pre-set attenuator. The main purpose of the attenuator was to balance out any loss occuring in the phase shifter,but it was also used to isolate one antenna when the radiation pattern of the other one alone was desired. The remaining arms of the magic-T were terminated by two crystal holders which in turn were connected to two audio amplifiers. The difference between the two audio outputs was then used to drive the microwave phase shifter, via a differential amplifier, servo amplifier and servo motor. The microwave assembly was mounted on the turntable of a conventional antenna pattern range, as shown in Figs. 11 and 12, and illuminated by a square wave modulated transmitter . Fig. 11 shows the front of the antennas with their feeds. The arrangement of the other microwave components may be seen in Fig. 12 .

The servo system is designed to equalise the two audio outputs. This in turn means equal outputs from the E and H arms of the magic-T or, in other words, a quadrature phase relationship between the two side arms. This quadrature condition would not of course be used in a normal receiving application since it splits the received

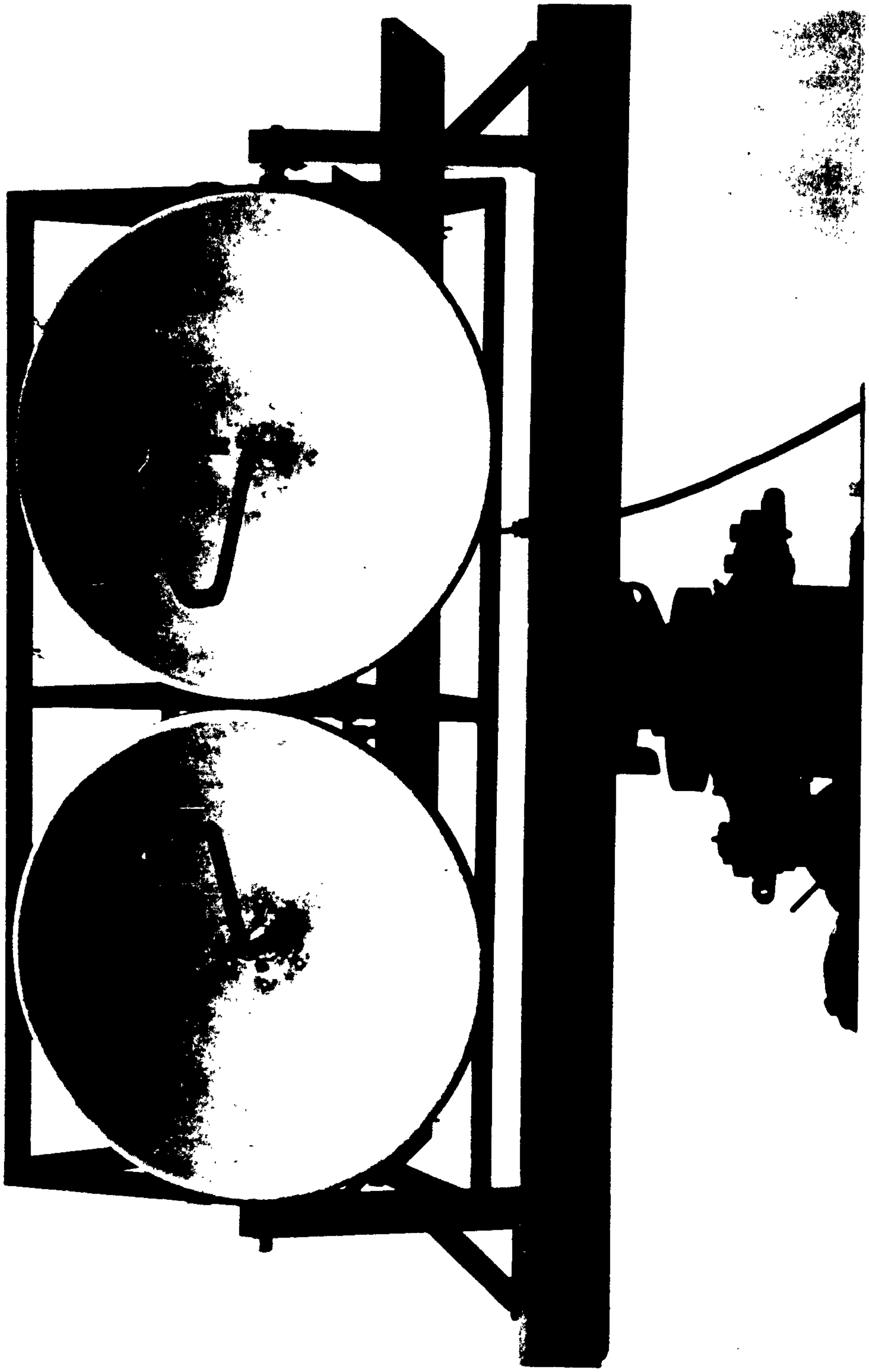

Fig. 11. Two element antenna system, front view.

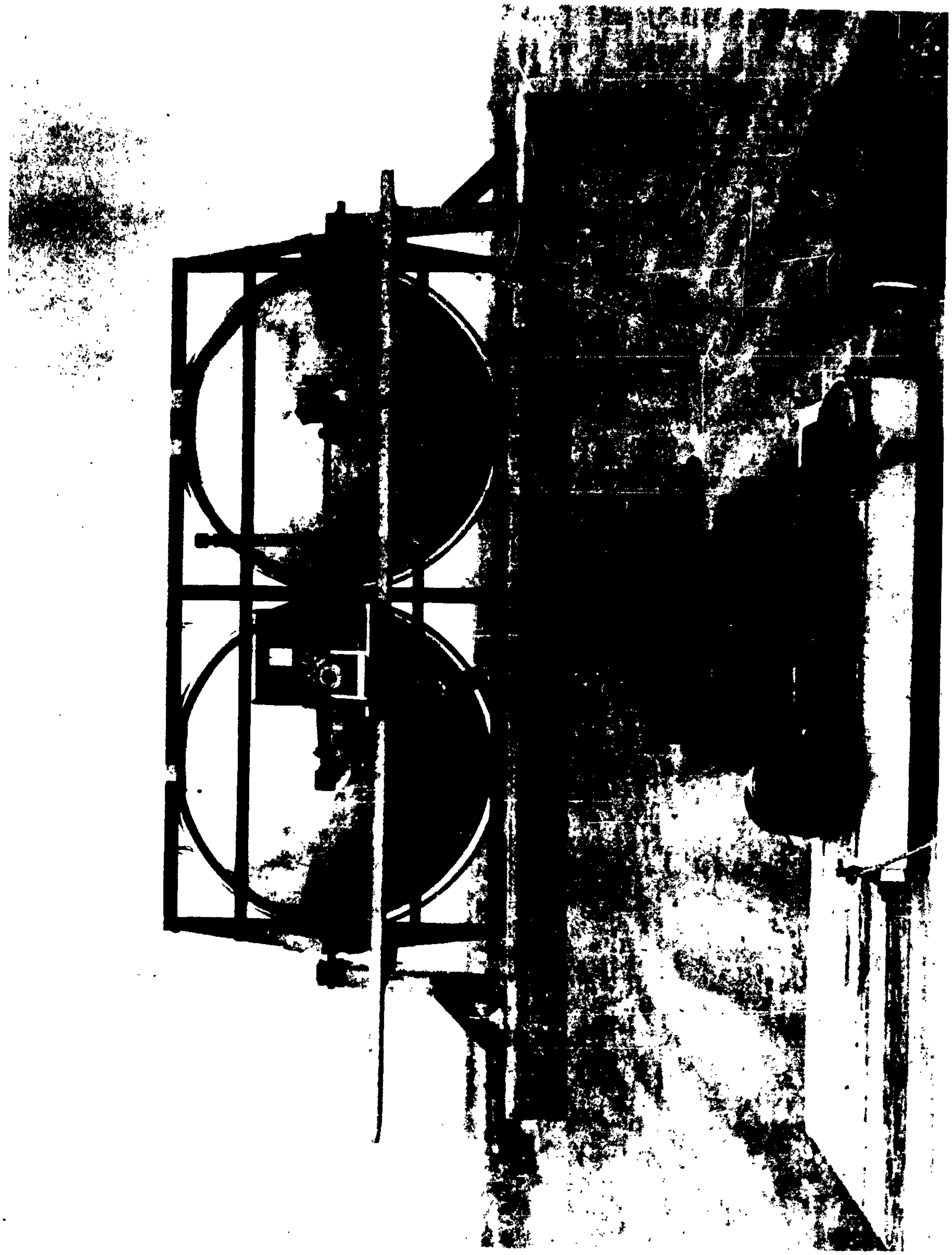

Fig. 12. Two element antenna system, back view.

power into two equal parts. However, it is convenient for the present purpose of radiation pattern measurements since it reduces the amount of equipment required. Since the two output powers are equal they are both directly related to the total received power. The logarithmic recorder, connected to the audio output from one arm , thus provides a direct measure of the received signal (Fig. 10) .

Initially the radiation pattern of one element alone was plotted by inserting maximum attenuation in the waveguide leading to the other element. This pattern is shown in Fig. 13(a) . Then, with both elements operative and the servo switched on, a second pattern was plotted as shown in Fig. 13(b) . It will be observed that the second pattern is substantially the same as the first except for an increase in gain of 3 db, as might be expected from double the aperture. However, it does not show any noticeable decrease in beamwidth, which would normally be associated with the increased aperture. Thus, in effect, the system provides a means of increasing antenna gain without decreasing beamwidth. It may thus be applied to situations when the received wavefront is not of constant phase such as those already discussed in section VII of this report. Of course it must be remembered that the pattern shown in Fig. 13(b) is the dynamic pattern of the complete system with the servo operating . It is in fact the envelope of all the different static patterns which would be obtained for various fixed positions of the phase shifter. One of these patterns for the case when the two antennas are in phase is shown in Fig. 13(c) .

XVI. AN X-BAND TRACKING RADIOMETER.

A general view of the radiometer is shown in Fig. 14 . It consists of two principal parts; the radiometer section and the tracking mechanism. The radiometer section is shown in schematic form in Fig. 15 . The antenna consists of a 48-inch diameter parabaloid fed by a small horn. The feed section may be detached just in front of the reflector to permit the use of alternate feeds or the connection of a noise source for calibration purposes. The antenna is connected to one arm of a four port ferrite switch. The opposite arm of the switch is connected to the reference load, to the outside of which a thermometer is attached so that the reference temperature may be accurately known. A third arm of the switch is connected to the input of a traveling-wave tube amplifier, the remaining arm being terminated in a dummy load for matching purposes. The output of the travelling wave-tube is connected to a crystal detector. All of these microwave components are mounted in a cylindrical housing directly behind the antenna (Fig. 14). They are so arranged that they may be rotated inside the housing, together with the antenna feed, in order to vary the plane of polarisation of the antenna. The output from the crystal detector is then connected to a narrow band audio frequency amplifier, located away from the antenna mount, the output of which is monitored on an

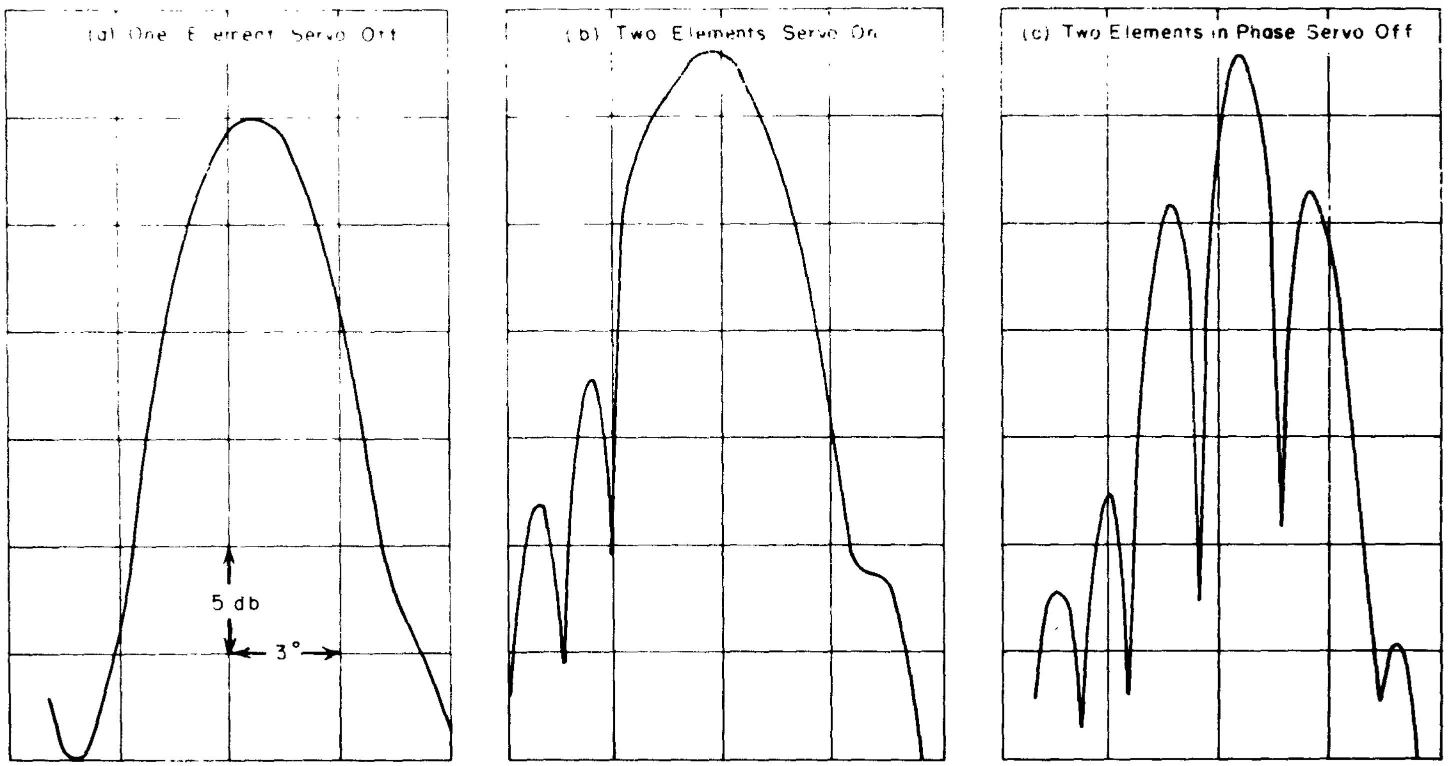

Fig. 13. Radiation patterns of a two element antenna .

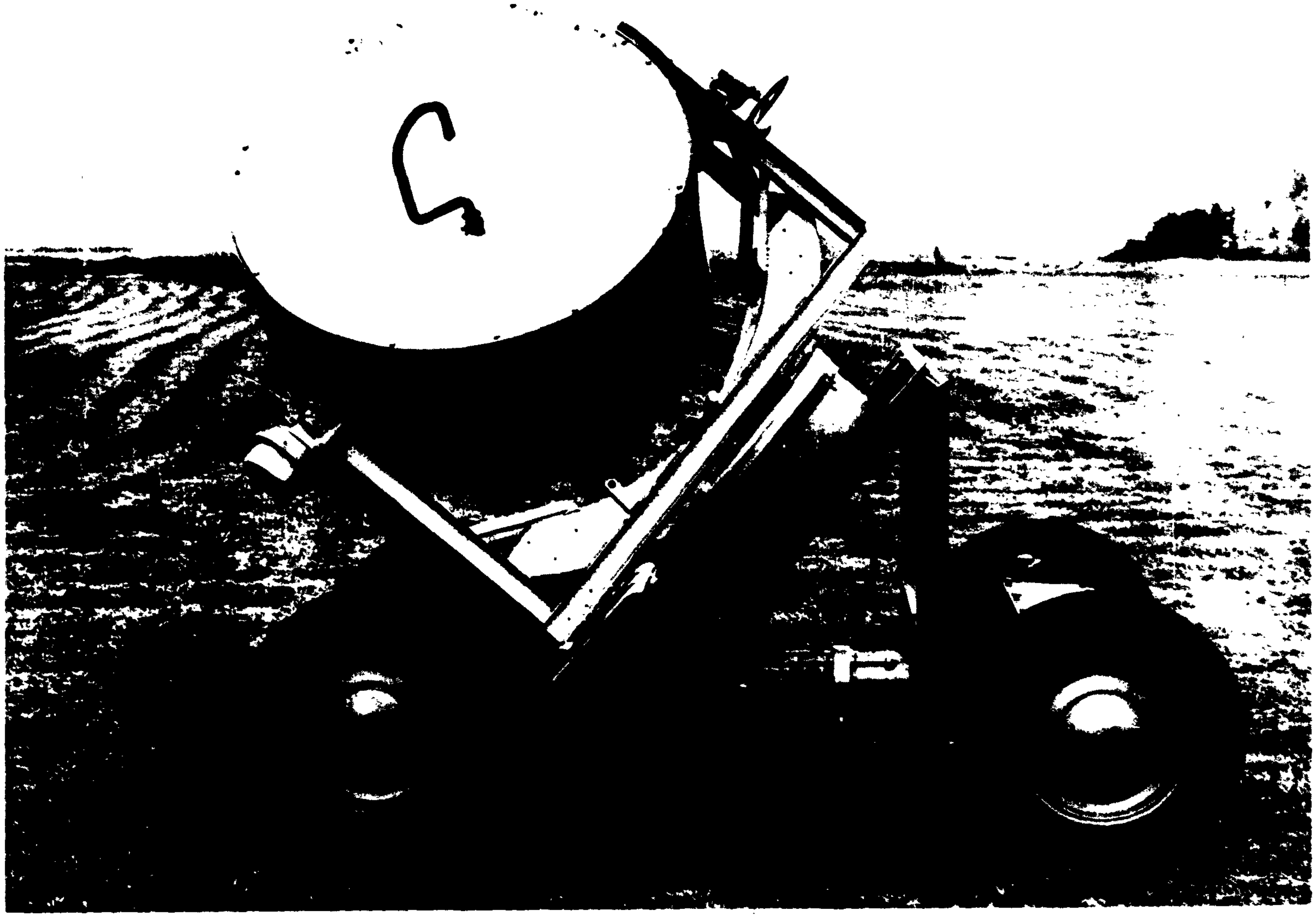

Fig. 14. An X-band solar tracking radiometer

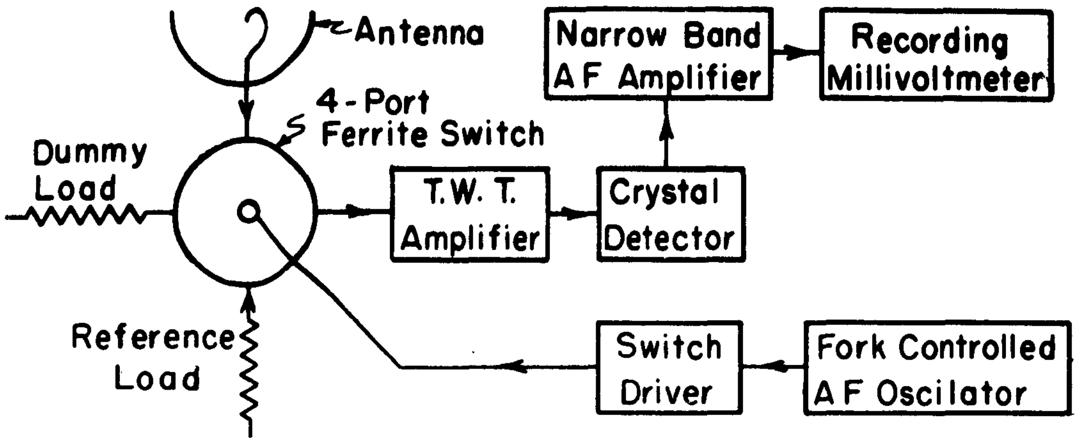

Fig. 15. Schematic diagram of X-band radiometer.

A.C. voltmeter. The ferrite switch is driven from an audio frequency power amplifier, the input to which is derived from an electrically maintained tuning fork tuned to the frequency of the narrow band amplifier. Because of the action of the ferrite switch the travelling wave tube alternately looks at the antenna and the reference load. The output from the crystal detector is thus approximately a square wave whose amplitude depends on the difference between the antenna and reference temperatures. In fact, since the crystal is inherently a square-law device, its output is directly proportional to power and hence to the difference temperature.

The tracking mechanism is shown in schematic form in Fig. 16. Since it is designed to track the sun it is geared to operate on solar time. However, a minor change of gear ratio in the clock drive mechanism would convert it to sidereal time, should this ever be required. The main chassis is adapted from that of a 60-inch military search light. The main turntable has been remounted at an angle of 50 degrees to the horizontal, the co-latitude of the locality in which the radiometer will be operated. Small variations in latitude may be accommodated by means of the four screw jacks located at the corners of the chassis. The turntable thus provides an hour angle motion when the center line of the chassis is aligned in a north-south direction. The radiometer is then mounted on this turntable with provision for manual declination adjustment by means of a handwheel and gear quadrant, which may be seen near the top of Fig. 14. Referring to Fig. 16, the turntable is driven by a quarter horsepower motor from

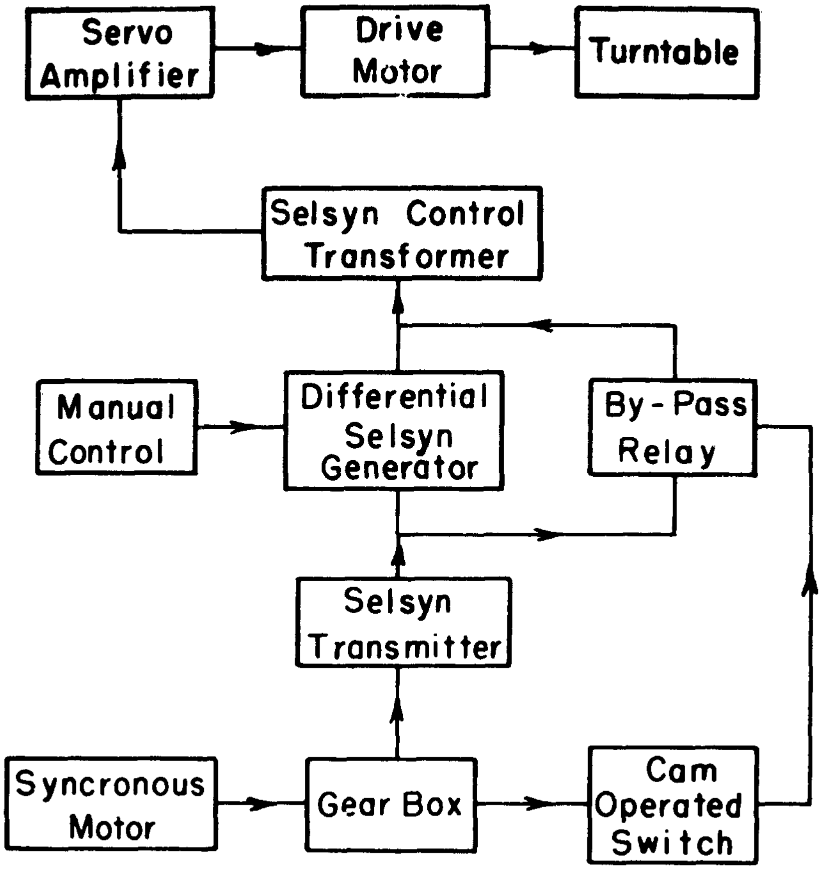

Fig. 16. Schematic diagram of radiometer tracking mechanism .

a selsyn control transformer, via a servo-amplifier and amplidyne. The control transformer receives its signal from a selsyn transmitter which is driven from a synchronous motor via a reduction gearbox . The gear ratios are adjusted so that the main turntable rotates once in twenty four hours. Thus, once the antenna has been aligned on the sun it should continue to follow it indefinitely. A differential selsyn may be inserted between the other two selsyn units. By manual adjustment of its shaft the antenna may be moved ahead or behind the sun's position without interfering with the time drive. This differential selsyn may be switched in or out of the circuit by means of a relay. The relay, in turn, may be actuated manually or by means of a cam operated microswitch located on the reduction gearbox. The microswitch is open and closed for equal periods of time, one complete cycle taking ten minutes. The antenna may thus be pointed at the sun for five minutes and then offset by some predetermined angle for the next five minutes and so on, the angle of offset being determined by the position of the differential selsyn.

XVII. RADIOMETER MEASUREMENTS.

The radiometer was first used to study antenna temperature as a function of environment and beam pointing angle. The results are illustrated in Fig. 1 which shows antenna temperature as a function of beam elevation angle for different types of foreground terrain and polarisation conditions. The temperatures are for a loss-less antenna, correction having been made for losses occuring in the feed system. In fact the temperature contribution due to feed loss for this particular antenna amounted to about 25°K and resulted, for the most part, from attenuation occuring in the waveguide bends. The readings were stable and showed good repeatability to an accuracy of better than $\pm$ 5°K. The stability was in part due to the narrow bandwidth of the audio amplifier providing an integration time of about a quarter of a second. Some long term drift was observed which required periodical checking of the calibration. This was due to gain fluctuation in the amplifiers and the open loop nature of the amplifying system. This presents some problems in the solar tracking experiments and modifications are now being made to use a variable noise source as a reference and to operate the amplifiers as a closed loop servo system.

Some further antenna temperature measurements were made with the reflector pointing at the sky and using a series of interchangeable feed horns. These were designed to study the effect of spill-over on antenna temperature. The same horns were then used in a similar reflector for gain measurements to provide data for optimising the signal-to-noise ratio of the antenna. These experiments are described in detail in reference 7.

The solar tracking experiments are designed to study the degree of phase distortion of a signal passing through the atmosphere, with the object of determining the limiting size of a single aperture antenna. The sun was chosen as the source, since none of the radio stars emit enough energy at X-band to be suitable, except with very large antennas. It has already been stated, in section VII of this report, that when the aperture size is increased the effect of phase distortion is first seen as a movement in the apparent position of the source. The magnitude of the distortion can thus be studied by measuring the extent of this movement. Ideally this requires a point source and a very narrow beam antenna. However, the sun cannot be regarded as a point source, being about half a degree in diameter. If a narrow beam antenna were used in this case any movement in the apparent position of the sun would only result in looking at a different part of the sun's surface resulting in little or no change in the observed temperature. In consequence an alternative approach is being tried in this case. The beamwidth of the present antenna is two degrees, between the half power points. By pointing the beam one degree away from the sun the

the latter will be located at the half pwer point on the steeply sloping side of the beam (Fig. 17(a)). Any small change in direction will

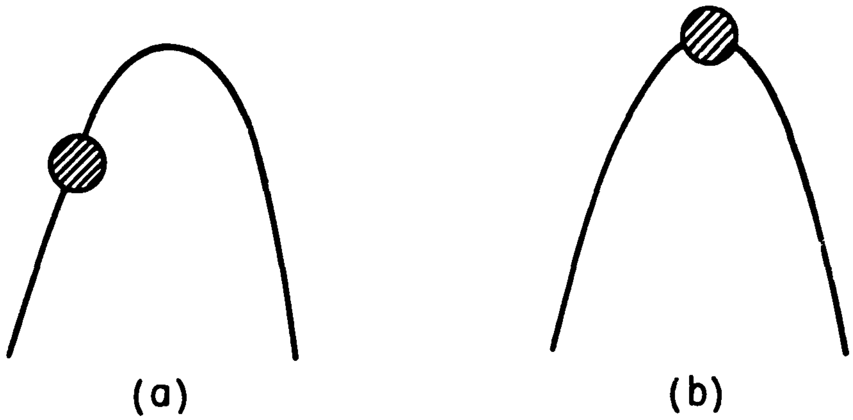

Fig. 17. Showing the effect of offsetting the antenna beam from the sun .

result in a movement of the sun up or down the slope and a consequent change in the antenna temperature. This change in temperature may be readilly calculated as follows. Assuming that the half power beam-width is 2^{o} and that the beam is cosine squared in shape the antenna temperature off axis, may be written as

(56) $$T_A = T'_A \cos^2 \left(\frac{x}{2} \cdot \frac{\pi}{2} \right)$$

where x is the angle from the axis in degrees and T'_A is the antenna temperature when the sun is on axis. It may be assumed for the moment that the sun is the only source of antenna noise since other sources are small by comparison. T'_A has been found experimentally to be approximately $500^{o}K$. Equation (56) may thus be written:

(57) $$T_A = 500 \cos^2 \frac{\pi x}{4}$$

$$\therefore \quad \frac{dT_A}{dx} = -500.2.\frac{\pi}{4} \cos \frac{\pi x}{4} \sin \frac{\pi x}{4}$$

(58) $$\therefore \quad = -125\,\pi \sin \frac{\pi x}{2}$$

This will be a maximum when x = 1, in other words at the half-power point.

$$\therefore \qquad \left(\frac{d\,T_A}{dx} \right)_{max} = -125\,\pi\ ^{o}K \text{ per degree of arc}$$

$$= \frac{-125\,\pi}{60}\ ^{o}K \text{ per minute of arc}$$

(59) $$\text{Temperature change} = 6.5^{o}K \text{ per minute of arc}$$

Given reasonable stability there should be no difficulty in observing a change of one minute of arc by this means. However, a change in the observed temperature could be the result of a change in the sun's temperature as well as of a phase disturbance . The sun's temperature is moderately constant at X-band, but small charges could well be observed. To eliminate this possibility the beam switching feature already described will be used. The antenna beam will be pointed directly at the sun for five minutes and then switched to the half power position for the next five and so on. With the beam pointed directly at the sun, Fig. 17(b), no temperature change should be observed as the result of changes in the apparent direction of the sun, since the latter's diameter is located on the flat top of the beam . Changes occuring in this period must, therefore, be due to charges in the sun's output. If variations are observed with the sun on the half power point but none are observed during the five minute periods immediately preceding and following it, then it is reasonable to assume that the variations are due to phase changes. Recordings over an extended period would, of course, strengthen this assumption . Recordings of this nature will shortly be made. The most interesting periods of the day will be just after sunrise and just before sunset when the sun's radiation has to traverse the greatest thickness of atmosphere. By studying these recordings it is hoped to obtain information which will determine the optimum size of elements for use in multielement antenna systems.

XVIII. CONCLUSIONS.

The most important result of this research program has been the generation of the concept of antenna gain-temperature ratio as a single design parameter. In the past antenna gain was used, but in low noise applications this is no longer adequate. The addition of antenna temperature filled the gap but introduced much tedious mathematics when the two quantities had to be handled separately . The present approach, using gain-temperature ratio as a single parameter eliminates this problem. A number of mathematical methods have been developed which will allow the designer to quickly optimise the performance of a given installation.

In addition, a number of factors limiting the performance of antennas have been considered including mechanical tollerances , phase distortion across the aperture and feed line losses. A multi-element antenna system has been suggested as a means of over-coming some of these problems. It has also been demonstrated that the use of amplifiers within the antenna system may help in some cases. However, it is concluded that the ultimate limitation on antenna performance is feedline loss. The maximum useful size of an antenna for long range communications is thus determined by the quality of the currently available transmission lines.

REFERENCES .

1. Kraus, J.D. and Ko, H.C., "Celestial Radio Radiation", Electrical Engineering Department, The Ohio State University, Project 673 .

2. Kraus, J.D., "Solar System Radio Radiation", Electrical Engineering Department, The Ohio State University, Project 673 .

3. Peake, W. and Taylor, R., "Apparent Antenna Temperature at L-Band Over Smooth Surfaces", Report 904-2, 15 December 1959 , (Antenna Laboratory, The Ohio State University Research Foundation, prepared under Contract DA-18-119-sc-533 U.S. Army , Signal Procurement Office, 9800 Savage Road, Fort George G. Meade, Maryland).

4. Chen, S.N.C., "Apparent Temperatures of Smooth and Rough Terrain", Report 898-8, 31 July 1960,(Antenna Laboratory, The Ohio State University Research Foundation ; prepared under Contract AF 33(616) 6158, Air Research and Development Command, Wright Air Development Division, Wright-Patterson Air Force Base, Ohio).

5. Cosgriff, R.L., Peake, W.H. and Taylor, R.C., "Terrain Scattering Properties for Sensor System Design" (Terrain Handbook II) Bulletin No. 181, Engineering Experiment Station, The Ohio State University .

6. Caldecott, R. and Brown, D., "The Design of Low Noise Antennas for Tropospheric Scatter Reception", Report 904-1, 15 January 1960, (Antenna Laboratory, The Ohio State University Research Foundation, prepared under Contract DA-18-119-sc-533 U. S. Army , Signal Procurement Office, 9800 Savage Road, Fort George G. Meade, Maryland).

7. Brown, D., "An Investigation of the Parameters of Parabaloidal Reflection Antennas Aimed at Securing Maximum Signal-to-Noise Ratio", Report 1041-2, 28 February 1961, (Antenna Laboratory ; The Ohio State University Research Foundation ; prepared under Contract AF 19(604) 6134, Air Force Cambridge Research Center, Air Research and Development Command, Laurence G. Hanscom Field , Bedford , Mass.)

8. Waterman, A.T., "A Rapid Beam Swinging Experiment", PGAP , AP-6, p. 338, 1958 .

9. Dwight, H.B., Tables of Integrals and Other Mathematical Data , p. 201, Macmillan .

NOTE: In submitting this report it is understood that all provisions of the contract between The Foundation and the Cooperator and pertaining to publicity of subject matter will be rigidly observed.

Investigator Ross Caldecott Date 3 March 61

Investigator .... Date ....

Investigator .... Date ....

Investigator .... Date ....

Supervisor T. E. Tice by R.C.T. Date 3 March 1961

For The Ohio State University Research Foundation

Executive Director Oran C. Woolpert W.C.H. Date 3 March 1961

1041-1